

or 
$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}. \quad (23-18)$$

This gives us

$$q' = q \frac{r^3}{R^3}. \quad (23-19)$$

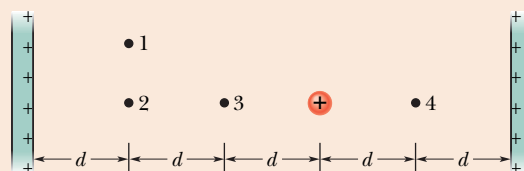
Substituting this into Eq. 23-17 yields

$$E = \left( \frac{q}{4\pi\epsilon_0 R^3} \right) r \quad (\text{uniform charge, field at } r \leq R). \quad (23-20)$$



#### CHECKPOINT 4

The figure shows two large, parallel, nonconducting sheets with identical (positive) uniform surface charge densities, and a sphere with a uniform (positive) volume charge density. Rank the four numbered points according to the magnitude of the net electric field there, greatest first.



## REVIEW & SUMMARY

**Gauss' Law** Gauss' law and Coulomb's law are different ways of describing the relation between charge and electric field in static situations. Gauss' law is

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (\text{Gauss' law}), \quad (23-6)$$

in which  $q_{\text{enc}}$  is the net charge inside an imaginary closed surface (a *Gaussian surface*) and  $\Phi$  is the net *flux* of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{electric flux through a Gaussian surface}). \quad (23-4)$$

Coulomb's law can be derived from Gauss' law.

**Applications of Gauss' Law** Using Gauss' law and, in some cases, symmetry arguments, we can derive several important results in electrostatic situations. Among these are:

1. An excess charge on an isolated *conductor* is located entirely on the outer surface of the conductor.
2. The external electric field near the *surface of a charged conductor* is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{conducting surface}). \quad (23-11)$$

Within the conductor,  $E = 0$ .

3. The electric field at any point due to an infinite *line of charge* with uniform linear charge density  $\lambda$  is perpendicular to the line of charge and has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge}), \quad (23-12)$$

where  $r$  is the perpendicular distance from the line of charge to the point.

4. The electric field due to an *infinite nonconducting sheet* with uniform surface charge density  $\sigma$  is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{sheet of charge}). \quad (23-13)$$

5. The electric field *outside a spherical shell of charge* with radius  $R$  and total charge  $q$  is directed radially and has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, for } r \geq R). \quad (23-15)$$

Here  $r$  is the distance from the center of the shell to the point at which  $E$  is measured. (The charge behaves, for external points, as if it were all located at the center of the sphere.) The field *inside* a uniform spherical shell of charge is exactly zero:

$$E = 0 \quad (\text{spherical shell, for } r < R). \quad (23-16)$$

6. The electric field *inside a uniform sphere of charge* is directed radially and has magnitude

$$E = \left( \frac{q}{4\pi\epsilon_0 R^3} \right) r. \quad (23-20)$$

## QUESTIONS

**1** A surface has the area vector  $\vec{A} = (2\hat{i} + 3\hat{j}) \text{ m}^2$ . What is the flux of a uniform electric field through the area if the field is (a)  $\vec{E} = 4\hat{i} \text{ N/C}$  and (b)  $\vec{E} = 4\hat{k} \text{ N/C}$ ?

**2** Figure 23-20 shows, in cross section, three solid cylinders, each of length  $L$  and uniform charge  $Q$ . Concentric with each cylinder is a cylindrical Gaussian surface, with all three surfaces having the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface, greatest first.

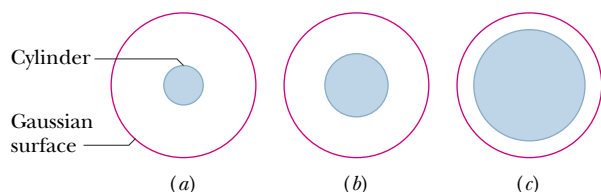


Fig. 23-20 Question 2.

**3** Figure 23-21 shows, in cross section, a central metal ball, two spherical metal shells, and three spherical Gaussian surfaces of radii  $R$ ,  $2R$ , and  $3R$ , all with the same center. The uniform charges on the three objects are: ball,  $Q$ ; smaller shell,  $3Q$ ; larger shell,  $5Q$ . Rank the Gaussian surfaces according to the magnitude of the electric field at any point on the surface, greatest first.

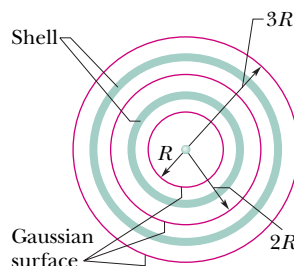


Fig. 23-21 Question 3.

**4** Figure 23-22 shows, in cross section, two Gaussian spheres and two Gaussian cubes that are centered on a positively charged particle. (a) Rank the net flux through the four Gaussian surfaces, greatest first. (b) Rank the magnitudes of the electric fields on the surfaces, greatest first, and indicate whether the magnitudes are uniform or variable along each surface.

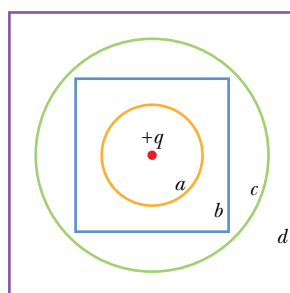


Fig. 23-22 Question 4.

**5** In Fig. 23-23, an electron is released between two infinite nonconducting sheets that are horizontal and have uniform surface charge densities  $\sigma_{(+)}$  and  $\sigma_{(-)}$ , as indicated. The electron is subjected to the following three situations involving surface charge densities and sheet separations. Rank the magnitudes of the electron's acceleration, greatest first.

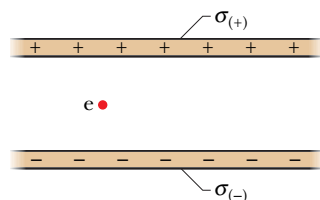


Fig. 23-23 Question 5.

Situation	$\sigma_{(+)}$	$\sigma_{(-)}$	Separation
1	$+4\sigma$	$-4\sigma$	$d$
2	$+7\sigma$	$-\sigma$	$4d$
3	$+3\sigma$	$-5\sigma$	$9d$

**6** Three infinite nonconducting sheets, with uniform positive surface charge densities  $\sigma$ ,  $2\sigma$ , and  $3\sigma$ , are arranged to be parallel like the two sheets in Fig. 23-17a. What is their order, from left to right, if the electric field  $\vec{E}$  produced by the arrangement has magnitude  $E = 0$  in one region and  $E = 2\sigma/\epsilon_0$  in another region?

**7** Figure 23-24 shows four situations in which four very long rods extend into and out of the page (we see only their cross sections). The value below each cross section gives that particular rod's uniform charge density in microcoulombs per meter. The rods are separated by either  $d$  or  $2d$  as drawn, and a central point is shown midway between the inner rods. Rank the situations according to the magnitude of the net electric field at that central point, greatest first.

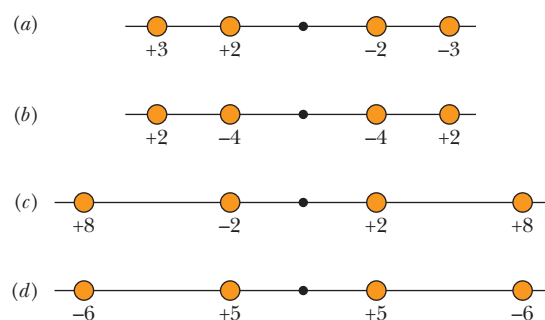


Fig. 23-24 Question 7.

**8** Figure 23-25 shows four solid spheres, each with charge  $Q$  uniformly distributed through its volume. (a) Rank the spheres according to their volume charge density, greatest first. The figure also shows a point  $P$  for each sphere, all at the same distance from the center of the sphere. (b) Rank the spheres according to the magnitude of the electric field they produce at point  $P$ , greatest first.

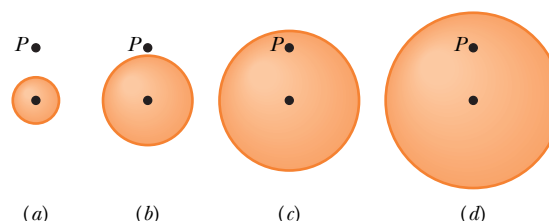


Fig. 23-25 Question 8.

**9** A small charged ball lies within the hollow of a metallic spherical shell of radius  $R$ . For three situations, the net charges on the ball and shell, respectively, are (1)  $+4q, 0$ ; (2)  $-6q, +10q$ ; (3)  $+16q, -12q$ . Rank the situations according to the charge on (a) the inner surface of the shell and (b) the outer surface, most positive first.

**10** Rank the situations of Question 9 according to the magnitude of the electric field (a) halfway through the shell and (b) at a point  $2R$  from the center of the shell, greatest first.



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at

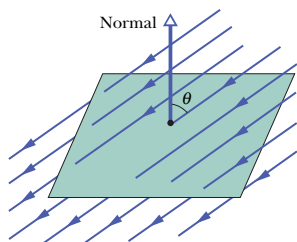


Number of dots indicates level of problem difficulty

ILW Interactive solution is at

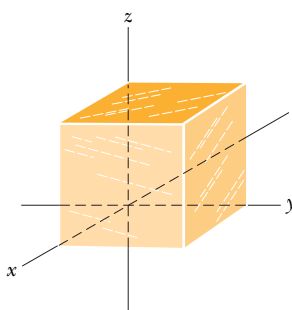
<http://www.wiley.com/college/halliday>Additional information available in *The Flying Circus of Physics* and at [flyingcircusofphysics.com](http://flyingcircusofphysics.com)**sec. 23-3 Flux of an Electric Field**

•1 **SSM** The square surface shown in Fig. 23-26 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude  $E = 1800 \text{ N/C}$  and with field lines at an angle of  $\theta = 35^\circ$  with a normal to the surface, as shown. Take that normal to be directed “outward,” as though the surface were one face of a box. Calculate the electric flux through the surface.

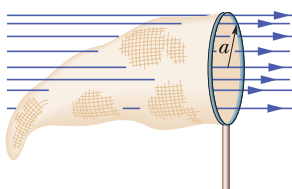
**Fig. 23-26** Problem 1.

•2 An electric field given by  $\vec{E} = 4.0\hat{i} - 3.0(y^2 + 2.0)\hat{j}$  pierces a Gaussian cube of edge length 2.0 m and positioned as shown in Fig. 23-5. (The magnitude  $E$  is in newtons per coulomb and the position  $x$  is in meters.) What is the electric flux through the (a) top face, (b) bottom face, (c) left face, and (d) back face? (e) What is the net electric flux through the cube?

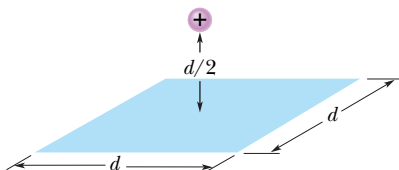
•3 The cube in Fig. 23-27 has edge length 1.40 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by (a)  $6.00\hat{i}$ , (b)  $-2.00\hat{j}$ , and (c)  $-3.00\hat{i} + 4.00\hat{k}$ . (d) What is the total flux through the cube for each field?

**Fig. 23-27** Problems 3, 6, and 9.**sec. 23-4 Gauss' Law**

•4 In Fig. 23-28, a butterfly net is in a uniform electric field of magnitude  $E = 3.0 \text{ mN/C}$ . The rim, a circle of radius  $a = 11 \text{ cm}$ , is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the netting.

**Fig. 23-28** Problem 4.

•5 In Fig. 23-29, a proton is a distance  $d/2$  directly above the center of a square of side  $d$ . What is the magnitude of the electric flux through the square? (*Hint*: Think of the square as one face of a cube with edge  $d$ .)

**Fig. 23-29** Problem 5.

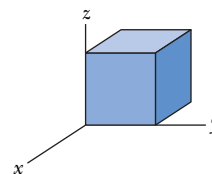
•6 At each point on the surface of the cube shown in Fig. 23-27, the electric field is parallel to the  $z$  axis. The length of each edge of the cube is 3.0 m. On the top face of the cube the field is  $\vec{E} = -34\hat{k} \text{ N/C}$ , and on the bottom face it is  $\vec{E} = +20\hat{k} \text{ N/C}$ . Determine the net charge contained within the cube.

•7 A point charge of  $1.8 \mu\text{C}$  is at the center of a Gaussian cube 55 cm on edge. What is the net electric flux through the surface?

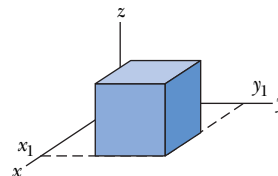
•8 When a shower is turned on in a closed bathroom, the splashing of the water on the bare tub can fill the room's air with negatively charged ions and produce an electric field in the air as great as  $1000 \text{ N/C}$ . Consider a bathroom with dimensions  $2.5 \text{ m} \times 3.0 \text{ m} \times 2.0 \text{ m}$ . Along the ceiling, floor, and four walls, approximate the electric field in the air as being directed perpendicular to the surface and as having a uniform magnitude of  $600 \text{ N/C}$ . Also, treat those surfaces as forming a closed Gaussian surface around the room's air. What are (a) the volume charge density  $\rho$  and (b) the number of excess elementary charges  $e$  per cubic meter in the room's air?

•9 **ILW** Fig. 23-27 shows a Gaussian surface in the shape of a cube with edge length 1.40 m. What are (a) the net flux  $\Phi$  through the surface and (b) the net charge  $q_{\text{enc}}$  enclosed by the surface if  $\vec{E} = (3.00y\hat{j}) \text{ N/C}$ , with  $y$  in meters? What are (c)  $\Phi$  and (d)  $q_{\text{enc}}$  if  $\vec{E} = [-4.00\hat{i} + (6.00 + 3.00y)\hat{j}] \text{ N/C}$ ?

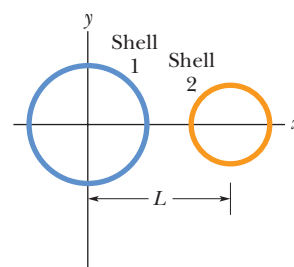
•10 Figure 23-30 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by  $\vec{E} = (3.00x + 4.00)\hat{i} + 6.00\hat{j} + 7.00\hat{k} \text{ N/C}$ , with  $x$  in meters. What is the net charge contained by the cube?

**Fig. 23-30** Problem 10.

•11 **ILW** Figure 23-31 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at  $x_1 = 5.00 \text{ m}$ ,  $y_1 = 4.00 \text{ m}$ . The cube lies in a region where the electric field vector is given by  $\vec{E} = -3.00\hat{i} - 4.00y^2\hat{j} + 3.00\hat{k} \text{ N/C}$ , with  $y$  in meters. What is the net charge contained by the cube?

**Fig. 23-31** Problem 11.

•12 Figure 23-32 shows two nonconducting spherical shells fixed in place. Shell 1 has uniform surface charge density  $+6.0 \mu\text{C/m}^2$  on its outer surface and radius 3.0 cm; shell 2 has uniform surface charge density  $+4.0 \mu\text{C/m}^2$  on its outer surface and radius 2.0 cm; the shell centers are separated by  $L = 10 \text{ cm}$ . In unit-vector notation, what is the net electric field at  $x = 2.0 \text{ cm}$ ?

**Fig. 23-32** Problem 12.

•13 **SSM** The electric field in a certain region of Earth's atmosphere is directed vertically down. At an altitude of 300 m the field

has magnitude  $60.0 \text{ N/C}$ ; at an altitude of  $200 \text{ m}$ , the magnitude is  $100 \text{ N/C}$ . Find the net amount of charge contained in a cube  $100 \text{ m}$  on edge, with horizontal faces at altitudes of  $200$  and  $300 \text{ m}$ .

••14 **Flux and nonconducting shells.** A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure 23-33a shows a cross section. Figure 23-33b gives the net flux  $\Phi$  through a Gaussian sphere centered on the particle, as a function of the radius  $r$  of the sphere. The scale of the vertical axis is set by  $\Phi_s = 5.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$ . (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?

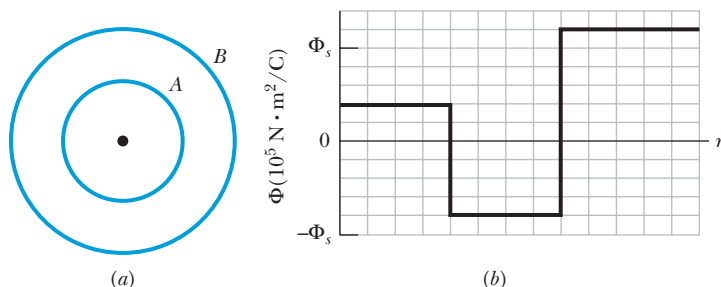


Fig. 23-33 Problem 14.

••15 A particle of charge  $+q$  is placed at one corner of a Gaussian cube. What multiple of  $q/\epsilon_0$  gives the flux through (a) each cube face forming that corner and (b) each of the other cube faces?

••16 **Box-like Gaussian surface.** The box-like Gaussian surface shown in Fig. 23-34 encloses a net charge of  $+24.0\epsilon_0 \text{ C}$  and lies in an electric field given by  $\vec{E} = [(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}] \text{ N/C}$ , with  $x$  and  $z$  in meters and  $b$  a constant. The bottom face is in the  $xz$  plane; the top face is in the horizontal plane passing through  $y_2 = 1.00 \text{ m}$ . For  $x_1 = 1.00 \text{ m}$ ,  $x_2 = 4.00 \text{ m}$ ,  $z_1 = 1.00 \text{ m}$ , and  $z_2 = 3.00 \text{ m}$ , what is  $b$ ?

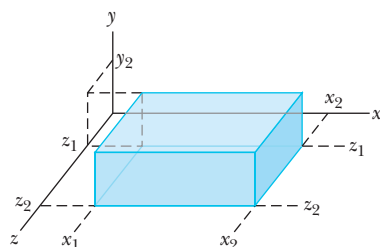


Fig. 23-34 Problem 16.

### sec. 23-6 A Charged Isolated Conductor

•17 **SSM** A uniformly charged conducting sphere of  $1.2 \text{ m}$  diameter has a surface charge density of  $8.1 \mu\text{C}/\text{m}^2$ . (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

•18 The electric field just above the surface of the charged conducting drum of a photocopying machine has a magnitude  $E$  of  $2.3 \times 10^5 \text{ N/C}$ . What is the surface charge density on the drum?

•19 Space vehicles traveling through Earth's radiation belts can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metal satellite  $1.3 \text{ m}$  in diameter accumulates  $2.4 \mu\text{C}$  of charge in one orbital revolution. (a) Find the resulting surface charge density. (b) Calculate the magnitude of the electric field just outside the surface of the satellite, due to the surface charge.

•20 **Flux and conducting shells.** A charged particle is held at the center of two concentric conducting spherical shells. Figure 23-35a shows a cross section. Figure 23-35b gives the net flux  $\Phi$  through a Gaussian sphere centered on the particle, as a function of the radius  $r$  of the sphere. The scale of the vertical axis is set by  $\Phi_s = 5.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$ . What are (a) the charge of the central particle and the net charges of (b) shell A and (c) shell B?

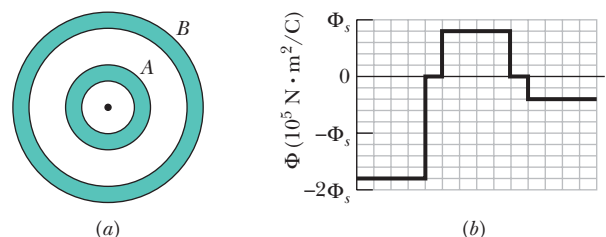


Fig. 23-35 Problem 20.

••21 An isolated conductor has net charge  $+10 \times 10^{-6} \text{ C}$  and a cavity with a point charge  $q = +3.0 \times 10^{-6} \text{ C}$ . What is the charge on (a) the cavity wall and (b) the outer surface?

### sec. 23-7 Applying Gauss' Law: Cylindrical Symmetry

•22 An electron is released  $9.0 \text{ cm}$  from a very long nonconducting rod with a uniform  $6.0 \mu\text{C}/\text{m}$ . What is the magnitude of the electron's initial acceleration?

•23 (a) The drum of a photocopying machine has a length of  $42 \text{ cm}$  and a diameter of  $12 \text{ cm}$ . The electric field just above the drum's surface is  $2.3 \times 10^5 \text{ N/C}$ . What is the total charge on the drum? (b) The manufacturer wishes to produce a desktop version of the machine. This requires reducing the drum length to  $28 \text{ cm}$  and the diameter to  $8.0 \text{ cm}$ . The electric field at the drum surface must not change. What must be the charge on this new drum?

•24 Figure 23-36 shows a section of a long, thin-walled metal tube of radius  $R = 3.00 \text{ cm}$ , with a charge per unit length of  $\lambda = 2.00 \times 10^{-8} \text{ C/m}$ . What is the magnitude  $E$  of the electric field at radial distance (a)  $r = R/2.00$  and (b)  $r = 2.00R$ ? (c) Graph  $E$  versus  $r$  for the range  $r = 0$  to  $2.00R$ .

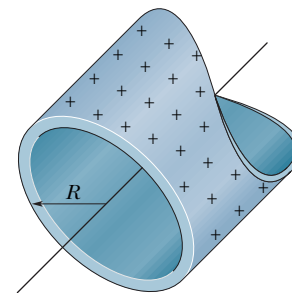


Fig. 23-36 Problem 24.

•25 **SSM** An infinite line of charge produces a field of magnitude  $4.5 \times 10^4 \text{ N/C}$  at distance  $2.0 \text{ m}$ . Find the linear charge density.

•26 Figure 23-37a shows a narrow charged solid cylinder that is coaxial with a larger charged cylindrical shell. Both are noncon-

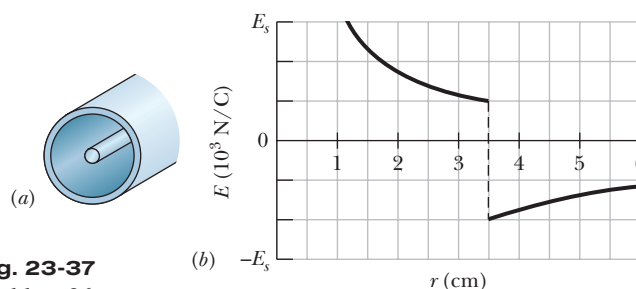


Fig. 23-37 Problem 26.



ducting and thin and have uniform surface charge densities on their outer surfaces. Figure 23-37b gives the radial component  $E$  of the electric field versus radial distance  $r$  from the common axis, and  $E_s = 3.0 \times 10^3$  N/C. What is the shell's linear charge density?

••27 A long, straight wire has fixed negative charge with a linear charge density of magnitude 3.6 nC/m. The wire is to be enclosed by a coaxial, thin-walled nonconducting cylindrical shell of radius 1.5 cm. The shell is to have positive charge on its outside surface with a surface charge density  $\sigma$  that makes the net external electric field zero. Calculate  $\sigma$ .

••28 **SSM** A charge of uniform linear density 2.0 nC/m is distributed along a long, thin, nonconducting rod. The rod is coaxial with a long conducting cylindrical shell (inner radius = 5.0 cm, outer radius = 10 cm). The net charge on the shell is zero. (a) What is the magnitude of the electric field 15 cm from the axis of the shell? What is the surface charge density on the (b) inner and (c) outer surface of the shell?

••29 **SSM WWW** Figure 23-38 is a section of a conducting rod of radius  $R_1 = 1.30$  mm and length  $L = 11.00$  m inside a thin-walled coaxial conducting cylindrical shell of radius  $R_2 = 10.0R_1$  and the (same) length  $L$ . The net charge on the rod is  $Q_1 = +3.40 \times 10^{-12}$  C; that on the shell is  $Q_2 = -2.00Q_1$ . What are the (a) magnitude  $E$  and (b) direction (radially inward or outward) of the electric field at radial distance  $r = 2.00R_2$ ? What are (c)  $E$  and (d) the direction at  $r = 5.00R_1$ ? What is the charge on the (e) interior and (f) exterior surface of the shell?

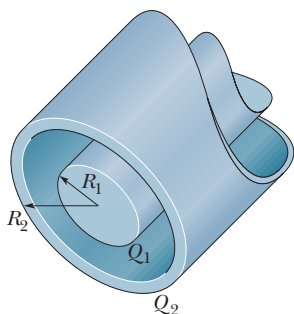


Fig. 23-38 Problem 29.

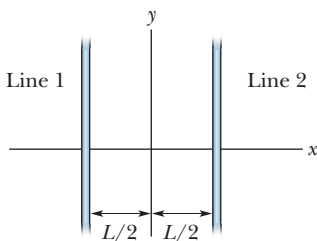


Fig. 23-39 Problem 30.

••30 In Fig. 23-39, short sections of two very long parallel lines of charge are shown, fixed in place, separated by  $L = 8.0$  cm. The uniform linear charge densities are  $+6.0 \mu\text{C/m}$  for line 1 and  $-2.0 \mu\text{C/m}$  for line 2. Where along the  $x$  axis shown is the net electric field from the two lines zero?

••31 **ILW** Two long, charged, thin-walled, concentric cylindrical shells have radii of 3.0 and 6.0 cm. The charge per unit length is  $5.0 \times 10^{-6}$  C/m on the inner shell and  $-7.0 \times 10^{-6}$  C/m on the outer shell. What are the (a) magnitude  $E$  and (b) direction (radially inward or outward) of the electric field at radial distance  $r = 4.0$  cm? What are (c)  $E$  and (d) the direction at  $r = 8.0$  cm?

••32 A long, nonconducting, solid cylinder of radius 4.0 cm has a nonuniform volume charge density  $\rho$  that is a function of radial distance  $r$  from the cylinder axis:  $\rho = Ar^2$ . For  $A = 2.5 \mu\text{C/m}^3$ , what is the magnitude of the electric field at (a)  $r = 3.0$  cm and (b)  $r = 5.0$  cm?

### sec. 23-8 Applying Gauss' Law: Planar Symmetry

•33 In Fig. 23-40, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge

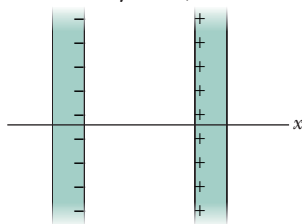


Fig. 23-40 Problem 33.

densities of opposite signs and magnitude  $7.00 \times 10^{-22}$  C/m<sup>2</sup>. In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them?

•34 In Fig. 23-41, a small circular hole of radius  $R = 1.80$  cm has been cut in the middle of an infinite, flat, nonconducting surface that has uniform charge density  $\sigma = 4.50$  pC/m<sup>2</sup>. A  $z$  axis, with its origin at the hole's center, is perpendicular to the surface. In unit-vector notation, what is the electric field at point  $P$  at  $z = 2.56$  cm? (Hint: See Eq. 22-26 and use superposition.)

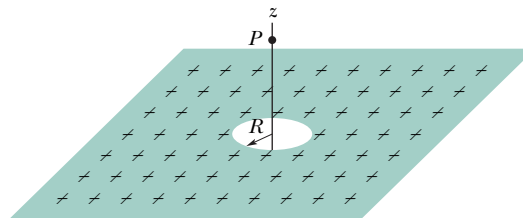


Fig. 23-41 Problem 34.

•35 **SSM** Figure 23-42a shows three plastic sheets that are large, parallel, and uniformly charged. Figure 23-42b gives the component of the net electric field along an  $x$  axis through the sheets. The scale of the vertical axis is set by  $E_s = 6.0 \times 10^5$  N/C. What is the ratio of the charge density on sheet 3 to that on sheet 2?

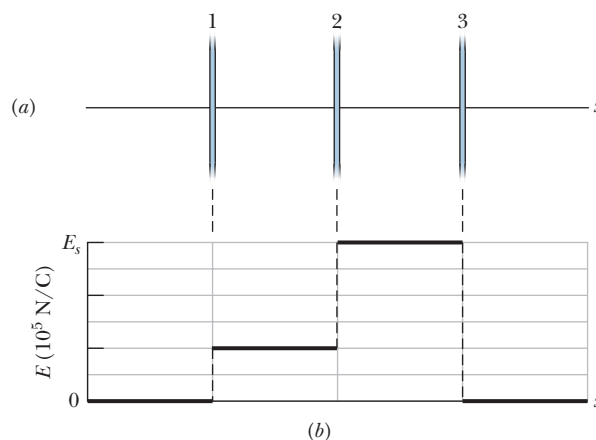


Fig. 23-42 Problem 35.

•36 Figure 23-43 shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density  $\sigma = 1.77 \times 10^{-22}$  C/m<sup>2</sup>. In unit-vector notation, what is  $\vec{E}$  at points (a) above the sheets, (b) between them, and (c) below them?

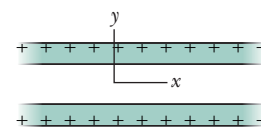


Fig. 23-43 Problem 36.

•37 **SSM WWW** A square metal plate of edge length 8.0 cm and negligible thickness has a total charge of  $6.0 \times 10^{-6}$  C. (a) Estimate the magnitude  $E$  of the electric field just off the center of the plate (at, say, a distance of 0.50 mm from the center) by assuming that the charge is spread uniformly over the two faces of the plate. (b) Estimate  $E$  at a distance of 30 m (large relative to the plate size) by assuming that the plate is a point charge.

••38 **SSM** In Fig. 23-44a, an electron is shot directly away from a uniformly charged plastic sheet, at speed  $v_s = 2.0 \times 10^5$  m/s. The sheet is

nonconducting, flat, and very large. Figure 23-44b gives the electron's vertical velocity component  $v$  versus time  $t$  until the return to the launch point. What is the sheet's surface charge density?

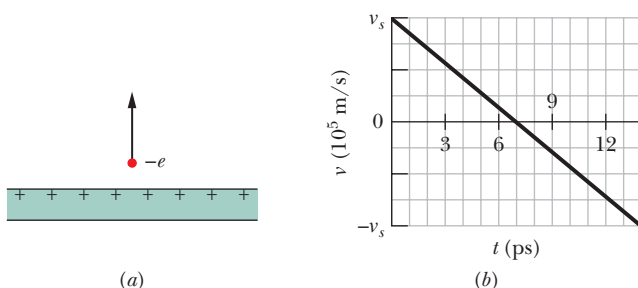


Fig. 23-44 Problem 38.

••39 **SSM** In Fig. 23-45, a small, nonconducting ball of mass  $m = 1.0$  mg and charge  $q = 2.0 \times 10^{-8}$  C (distributed uniformly through its volume) hangs from an insulating thread that makes an angle  $\theta = 30^\circ$  with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density  $\sigma$  of the sheet.

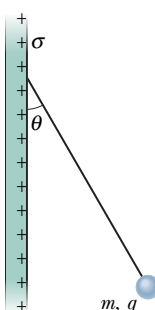


Fig. 23-45 Problem 39.

••40 Figure 23-46 shows a very large nonconducting sheet that has a uniform surface charge density of  $\sigma = -2.00 \mu\text{C}/\text{m}^2$ ; it also shows a particle of charge  $Q = 6.00 \mu\text{C}$ , at distance  $d$  from the sheet. Both are fixed in place. If  $d = 0.200$  m, at what (a) positive and (b) negative coordinate on the  $x$  axis (other than infinity) is the net electric field  $\vec{E}_{\text{net}}$  of the sheet and particle zero? (c) If  $d = 0.800$  m, at what coordinate on the  $x$  axis is  $\vec{E}_{\text{net}} = 0$ ?

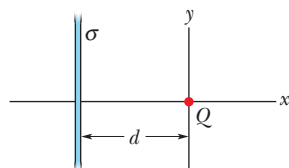


Fig. 23-46 Problem 40.

••41 **SSM** An electron is shot directly toward the center of a large metal plate that has surface charge density  $-2.0 \times 10^{-6} \text{ C}/\text{m}^2$ . If the initial kinetic energy of the electron is  $1.60 \times 10^{-17}$  J and if the electron is to stop (due to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must the launch point be?

••42 Two large metal plates of area  $1.0 \text{ m}^2$  face each other, 5.0 cm apart, with equal charge magnitudes  $|q|$  but opposite signs. The field magnitude  $E$  between them (neglect fringing) is 55 N/C. Find  $|q|$ .

••43 Figure 23-47 shows a cross section through a very large nonconducting slab of thickness  $d = 9.40$  mm and uniform volume charge density  $\rho = 5.80 \text{ fC}/\text{m}^3$ . The origin of an  $x$  axis is at the slab's center. What is the magnitude of the slab's electric field at an  $x$  coordinate of (a) 0, (b) 2.00 mm, (c) 4.70 mm, and (d) 26.0 mm?

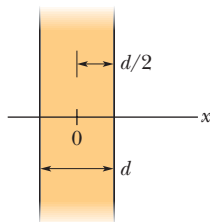


Fig. 23-47 Problem 43.

### sec. 23-9 Applying Gauss' Law: Spherical Symmetry

•44 Figure 23-48 gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly through-

out its volume. The scale of the vertical axis is set by  $E_s = 5.0 \times 10^7 \text{ N/C}$ . What is the charge on the sphere?

•45 Two charged concentric spherical shells have radii 10.0 cm and 15.0 cm. The charge on the inner shell is  $4.00 \times 10^{-8} \text{ C}$ , and that on the outer shell is  $2.00 \times 10^{-8} \text{ C}$ . Find the electric field (a) at  $r = 12.0$  cm and (b) at  $r = 20.0$  cm.

•46 A point charge causes an electric flux of  $-750 \text{ N} \cdot \text{m}^2/\text{C}$  to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

•47 **SSM** An unknown charge sits on a conducting solid sphere of radius 10 cm. If the electric field 15 cm from the center of the sphere has the magnitude  $3.0 \times 10^3 \text{ N/C}$  and is directed radially inward, what is the net charge on the sphere?

••48 A charged particle is held at the center of a spherical shell. Figure 23-49 gives the magnitude  $E$  of the electric field versus radial distance  $r$ . The scale of the vertical axis is set by  $E_s = 10.0 \times 10^7 \text{ N/C}$ . Approximately, what is the net charge on the shell?

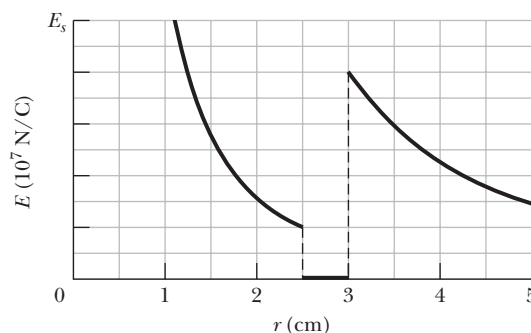


Fig. 23-49 Problem 48.

••49 In Fig. 23-50, a solid sphere of radius  $a = 2.00$  cm is concentric with a spherical conducting shell of inner radius  $b = 2.00a$  and outer radius  $c = 2.40a$ . The sphere has a net uniform charge  $q_1 = +5.00 \text{ fC}$ ; the shell has a net charge  $q_2 = -q_1$ . What is the magnitude of the electric field at radial distances (a)  $r = 0$ , (b)  $r = a/2.00$ , (c)  $r = a$ , (d)  $r = 1.50a$ , (e)  $r = 2.30a$ , and (f)  $r = 3.50a$ ? What is the net charge on the (g) inner and (h) outer surface of the shell?

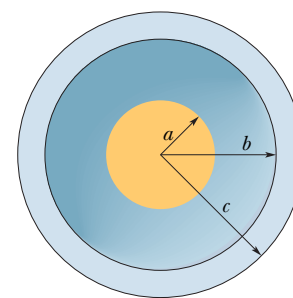


Fig. 23-50 Problem 49.

••50 **SSM** Figure 23-51 shows two nonconducting spherical shells fixed in place on an  $x$  axis. Shell 1 has uniform surface charge density  $+4.0 \mu\text{C}/\text{m}^2$  on its outer surface and radius 0.50 cm, and shell 2 has uniform surface charge density  $-2.0 \mu\text{C}/\text{m}^2$  on its outer surface and radius 2.0 cm; the centers are separated by  $L = 6.0$  cm. Other than at  $x = \infty$ , where on the  $x$  axis is the net electric field equal to zero?

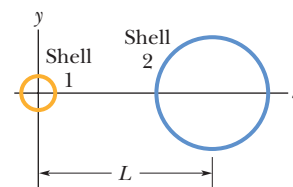


Fig. 23-51 Problem 50.

**•51 SSM WWW** In Fig. 23-52, a nonconducting spherical shell of inner radius  $a = 2.00$  cm and outer radius  $b = 2.40$  cm has (within its thickness) a positive volume charge density  $\rho = A/r$ , where  $A$  is a constant and  $r$  is the distance from the center of the shell. In addition, a small ball of charge  $q = 45.0$  fC is located at that center. What value should  $A$  have if the electric field in the shell ( $a \leq r \leq b$ ) is to be uniform?

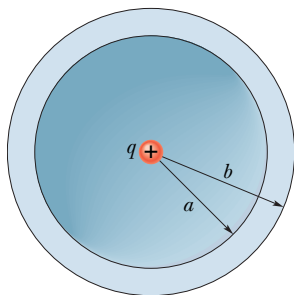


Fig. 23-52 Problem 51.

**•52** Figure 23-53 shows a spherical shell with uniform volume charge density  $\rho = 1.84$  nC/m<sup>3</sup>, inner radius  $a = 10.0$  cm, and outer radius  $b = 2.00a$ . What is the magnitude of the electric field at radial distances (a)  $r = 0$ ; (b)  $r = a/2.00$ , (c)  $r = a$ , (d)  $r = 1.50a$ , (e)  $r = b$ , and (f)  $r = 3.00b$ ?

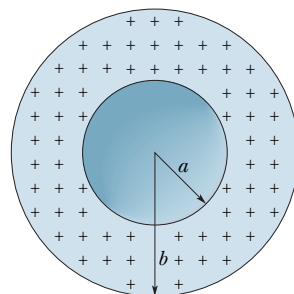


Fig. 23-53 Problem 52.

**•53 ILW** The volume charge density of a solid nonconducting sphere of radius  $R = 5.60$  cm varies with radial distance  $r$  as given by  $\rho = (14.1 \text{ pC/m}^3)r/R$ . (a) What is the sphere's total charge? What is the field magnitude  $E$  at (b)  $r = 0$ , (c)  $r = R/2.00$ , and (d)  $r = R$ ? (e) Graph  $E$  versus  $r$ .

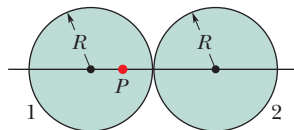


Fig. 23-54 Problem 54.

**•54** Figure 23-54 shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius  $R$ . Point  $P$  lies on a line connecting the centers of the spheres, at radial distance  $R/2.00$  from the center of sphere 1. If the net electric field at point  $P$  is zero, what is the ratio  $q_2/q_1$  of the total charges?

**•55** A charge distribution that is spherically symmetric but not uniform radially produces an electric field of magnitude  $E = Kr^4$ , directed radially outward from the center of the sphere. Here  $r$  is the radial distance from that center, and  $K$  is a constant. What is the volume density  $\rho$  of the charge distribution?

### Additional Problems

**56** The electric field in a particular space is  $\vec{E} = (x + 2)\hat{i}$  N/C, with  $x$  in meters. Consider a cylindrical Gaussian surface of radius 20 cm that is coaxial with the  $x$  axis. One end of the cylinder is at  $x = 0$ . (a) What is the magnitude of the electric flux through the other end of the cylinder at  $x = 2.0$  m? (b) What net charge is enclosed within the cylinder?

**57** A thin-walled metal spherical shell has radius 25.0 cm and charge  $2.00 \times 10^{-7}$  C. Find  $E$  for a point (a) inside the shell, (b) just outside it, and (c) 3.00 m from the center.

**58** A uniform surface charge of density 8.0 nC/m<sup>2</sup> is distributed over the entire  $xy$  plane. What is the electric flux through a spherical Gaussian surface centered on the origin and having a radius of 5.0 cm?

**59** Charge of uniform volume density  $\rho = 1.2$  nC/m<sup>3</sup> fills an infinite slab between  $x = -5.0$  cm and  $x = +5.0$  cm. What is the magnitude of the electric field at any point with the coordinate (a)  $x = 4.0$  cm and (b)  $x = 6.0$  cm?

**60** *The chocolate crumb mystery.* Explosions ignited by electrostatic discharges (sparks) constitute a serious danger in facilities handling grain or powder. Such an explosion occurred in chocolate crumb powder at a biscuit factory in the 1970s. Workers usually emptied newly delivered sacks of the powder into a loading bin, from which it was blown through electrically grounded plastic pipes to a silo for storage. Somewhere along this route, two conditions for an explosion were met: (1) The magnitude of an electric field became  $3.0 \times 10^6$  N/C or greater, so that electrical breakdown and thus sparking could occur. (2) The energy of a spark was 150 mJ or greater so that it could ignite the powder explosively. Let us check for the first condition in the powder flow through the plastic pipes.

Suppose a stream of *negatively* charged powder was blown through a cylindrical pipe of radius  $R = 5.0$  cm. Assume that the powder and its charge were spread uniformly through the pipe with a volume charge density  $\rho$ . (a) Using Gauss' law, find an expression for the magnitude of the electric field  $\vec{E}$  in the pipe as a function of radial distance  $r$  from the pipe center. (b) Does  $E$  increase or decrease with increasing  $r$ ? (c) Is  $\vec{E}$  directed radially inward or outward? (d) For  $\rho = 1.1 \times 10^{-3}$  C/m<sup>3</sup> (a typical value at the factory), find the maximum  $E$  and determine where that maximum field occurs. (e) Could sparking occur, and if so, where? (The story continues with Problem 70 in Chapter 24.)

**61 SSM** A thin-walled metal spherical shell of radius  $a$  has a charge  $q_a$ . Concentric with it is a thin-walled metal spherical shell of radius  $b > a$  and charge  $q_b$ . Find the electric field at points a distance  $r$  from the common center, where (a)  $r < a$ , (b)  $a < r < b$ , and (c)  $r > b$ . (d) Discuss the criterion you would use to determine how the charges are distributed on the inner and outer surfaces of the shells.

**62** A point charge  $q = 1.0 \times 10^{-7}$  C is at the center of a spherical cavity of radius 3.0 cm in a chunk of metal. Find the electric field (a) 1.5 cm from the cavity center and (b) anywhere in the metal.

**63** A proton at speed  $v = 3.00 \times 10^5$  m/s orbits at radius  $r = 1.00$  cm outside a charged sphere. Find the sphere's charge.

**64** Equation 23-11 ( $E = \sigma/\epsilon_0$ ) gives the electric field at points near a charged conducting surface. Apply this equation to a conducting sphere of radius  $r$  and charge  $q$ , and show that the electric field outside the sphere is the same as the field of a point charge located at the center of the sphere.

**65** Charge  $Q$  is uniformly distributed in a sphere of radius  $R$ . (a) What fraction of the charge is contained within the radius  $r = R/2.00$ ? (b) What is the ratio of the electric field magnitude at  $r = R/2.00$  to that on the surface of the sphere?

**66** Assume that a ball of charged particles has a uniformly distributed negative charge density except for a narrow radial tunnel through its center, from the surface on one side to the surface on the opposite side. Also assume that we can position a proton anywhere along the tunnel or outside the ball. Let  $F_R$  be the magnitude of the electrostatic force on the proton when it is located at the ball's surface, at radius  $R$ . As a multiple of  $R$ , how far from the surface is there a point where the force magnitude is  $0.50F_R$  if we move the proton (a) away from the ball and (b) into the tunnel?

**67 SSM** The electric field at point  $P$  just outside the outer surface of a hollow spherical conductor of inner radius 10 cm and outer radius 20 cm has magnitude 450 N/C and is directed outward. When an unknown point charge  $Q$  is introduced into the center of the sphere, the electric field at  $P$  is still directed outward but is now 180 N/C. (a) What was the net charge enclosed by the



outer surface before  $Q$  was introduced? (b) What is charge  $Q$ ? After  $Q$  is introduced, what is the charge on the (c) inner and (d) outer surface of the conductor?

**68** The net electric flux through each face of a die (singular of dice) has a magnitude in units of  $10^3 \text{ N} \cdot \text{m}^2/\text{C}$  that is exactly equal to the number of spots  $N$  on the face (1 through 6). The flux is inward for  $N$  odd and outward for  $N$  even. What is the net charge inside the die?

**69** Figure 23-55 shows, in cross section, three infinitely large nonconducting sheets on which charge is uniformly spread. The surface charge densities are  $\sigma_1 = +2.00 \mu\text{C}/\text{m}^2$ ,  $\sigma_2 = +4.00 \mu\text{C}/\text{m}^2$ , and  $\sigma_3 = -5.00 \mu\text{C}/\text{m}^2$ , and distance  $L = 1.50 \text{ cm}$ . In unit-vector notation, what is the net electric field at point  $P$ ?

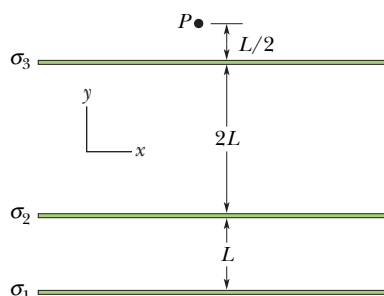


Fig. 23-55 Problem 69.

**70** Charge of uniform volume density  $\rho = 3.2 \mu\text{C}/\text{m}^3$  fills a nonconducting solid sphere of radius  $5.0 \text{ cm}$ . What is the magnitude of the electric field (a)  $3.5 \text{ cm}$  and (b)  $8.0 \text{ cm}$  from the sphere's center?

**71** A Gaussian surface in the form of a hemisphere of radius  $R = 5.68 \text{ cm}$  lies in a uniform electric field of magnitude  $E = 2.50 \text{ N/C}$ . The surface encloses no net charge. At the (flat) base of the surface, the field is perpendicular to the surface and directed into the surface. What is the flux through (a) the base and (b) the curved portion of the surface?

**72** What net charge is enclosed by the Gaussian cube of Problem 2?

**73** A nonconducting solid sphere has a uniform volume charge density  $\rho$ . Let  $\vec{r}$  be the vector from the center of the sphere to a general point  $P$  within the sphere. (a) Show that the electric field at  $P$  is given by  $\vec{E} = \rho\vec{r}/3\epsilon_0$ . (Note that the result is independent of the radius of the sphere.) (b) A spherical cavity is hollowed out of the sphere, as shown in Fig. 23-56. Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to  $\vec{E} = \rho\vec{a}/3\epsilon_0$ , where  $\vec{a}$  is the position vector from the center of the sphere to the center of the cavity.

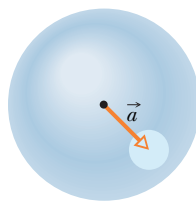


Fig. 23-56 Problem 73.

**74** A uniform charge density of  $500 \text{ nC}/\text{m}^3$  is distributed throughout a spherical volume of radius  $6.00 \text{ cm}$ . Consider a cubical Gaussian surface with its center at the center of the sphere. What is the electric flux through this cubical surface if its edge length is (a)  $4.00 \text{ cm}$  and (b)  $14.0 \text{ cm}$ ?

**75** Figure 23-57 shows a Geiger counter, a device used to detect ionizing radiation, which causes ionization of atoms. A thin, positively

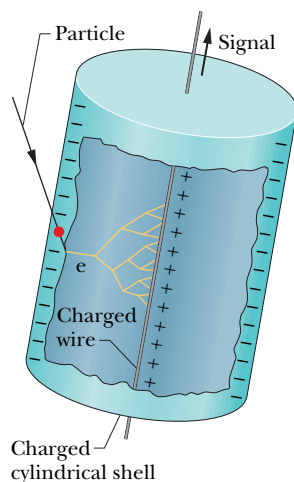


Fig. 23-57 Problem 75.

charged central wire is surrounded by a concentric, circular, conducting cylindrical shell with an equal negative charge, creating a strong radial electric field. The shell contains a low-pressure inert gas. A particle of radiation entering the device through the shell wall ionizes a few of the gas atoms. The resulting free electrons ( $e$ ) are drawn to the positive wire. However, the electric field is so intense that, between collisions with gas atoms, the free electrons gain energy sufficient to ionize these atoms also. More free electrons are thereby created, and the process is repeated until the electrons reach the wire. The resulting "avalanche" of electrons is collected by the wire, generating a signal that is used to record the passage of the original particle of radiation. Suppose that the radius of the central wire is  $25 \mu\text{m}$ , the inner radius of the shell  $1.4 \text{ cm}$ , and the length of the shell  $16 \text{ cm}$ . If the electric field at the shell's inner wall is  $2.9 \times 10^4 \text{ N/C}$ , what is the total positive charge on the central wire?

**76** Charge is distributed uniformly throughout the volume of an infinitely long solid cylinder of radius  $R$ . (a) Show that, at a distance  $r < R$  from the cylinder axis,

$$E = \frac{\rho r}{2\epsilon_0},$$

where  $\rho$  is the volume charge density. (b) Write an expression for  $E$  when  $r > R$ .

**77 SSM** A spherical conducting shell has a charge of  $-14 \mu\text{C}$  on its outer surface and a charged particle in its hollow. If the net charge on the shell is  $-10 \mu\text{C}$ , what is the charge (a) on the inner surface of the shell and (b) of the particle?

**78** A charge of  $6.00 \text{ pC}$  is spread uniformly throughout the volume of a sphere of radius  $r = 4.00 \text{ cm}$ . What is the magnitude of the electric field at a radial distance of (a)  $6.00 \text{ cm}$  and (b)  $3.00 \text{ cm}$ ?

**79** Water in an irrigation ditch of width  $w = 3.22 \text{ m}$  and depth  $d = 1.04 \text{ m}$  flows with a speed of  $0.207 \text{ m/s}$ . The mass flux of the flowing water through an imaginary surface is the product of the water's density ( $1000 \text{ kg}/\text{m}^3$ ) and its volume flux through that surface. Find the mass flux through the following imaginary surfaces: (a) a surface of area  $wd$ , entirely in the water, perpendicular to the flow; (b) a surface with area  $3wd/2$ , of which  $wd$  is in the water, perpendicular to the flow; (c) a surface of area  $wd/2$ , entirely in the water, perpendicular to the flow; (d) a surface of area  $wd$ , half in the water and half out, perpendicular to the flow; (e) a surface of area  $wd$ , entirely in the water, with its normal  $34.0^\circ$  from the direction of flow.

**80** Charge of uniform surface density  $8.00 \text{ nC}/\text{m}^2$  is distributed over an entire  $xy$  plane; charge of uniform surface density  $3.00 \text{ nC}/\text{m}^2$  is distributed over the parallel plane defined by  $z = 2.00 \text{ m}$ . Determine the magnitude of the electric field at any point having a  $z$  coordinate of (a)  $1.00 \text{ m}$  and (b)  $3.00 \text{ m}$ .

**81** A spherical ball of charged particles has a uniform charge density. In terms of the ball's radius  $R$ , at what radial distances (a) inside and (b) outside the ball is the magnitude of the ball's electric field equal to  $\frac{1}{4}$  of the maximum magnitude of that field?

**82 SSM** A free electron is placed between two large, parallel, nonconducting plates that are horizontal and  $2.3 \text{ cm}$  apart. One plate has a uniform positive charge; the other has an equal amount of uniform negative charge. The force on the electron due to the electric field  $\vec{E}$  between the plates balances the gravitational force on the electron. What are (a) the magnitude of the surface charge density on the plates and (b) the direction (up or down) of  $\vec{E}$ ?